Semi classical nuclear level density in the micro-macroscopic approach

A.G. Magner,[1] A.I. Sanzhur,[1] S.N. Fedotkin,[1] A.I. Levon,[1] U.V. Grygoriev,[1,2] and S. Shlomo
[1]*Institute for Nuclear Research, National Academy of Sciences of Ukraine, Kyiv, Ukraine*
[2]*Faculty of Science and Engineering, University of Groningen, Groningen, Netherlands*

Many properties of atomic nuclei were described in terms of the statistical level density, ρ, for large excitation energies U at a thermal equilibrium with temperature T [1]. The nuclear level density ρ(E,**Q**) was derived within the micro-macroscopic approximation (MMA) for a system of strongly interacting nucleons with the energy E and additional integrals of motion **Q** [2,3]. Within the extended Thomas-Fermi (ETF) approach and semiclassical periodic-orbit theory (POT), beyond the Fermi-gas saddle-point method we obtain $\rho_\Lambda \approx \bar{\rho} I_\nu(S)/S^\nu$,where I_ν is the modified Bessel function of the entropy S of order ν, and $\bar{\rho}$ is a constant independent of S. For small shell-structure contribution, one finds $\nu = \kappa/2 + 1$, where κ is the number of additional integrals of motion. This integer number is of a dimension of **Q**, **Q** = (*N,Z,*…) for the case of atomic nuclei. Here, N and Z are the numbers of neutrons and protons, respectively. For much larger shell structure contributions, one obtains $\nu=\kappa/2+2$. The MMA level density ρ reaches the well-known Fermi gas asymptote [1] for large entropy S (large excitation energy U), and the finite micro-canonical combinatoric limit for low entropy S (low excitation energies U→0), see Fig. 1.

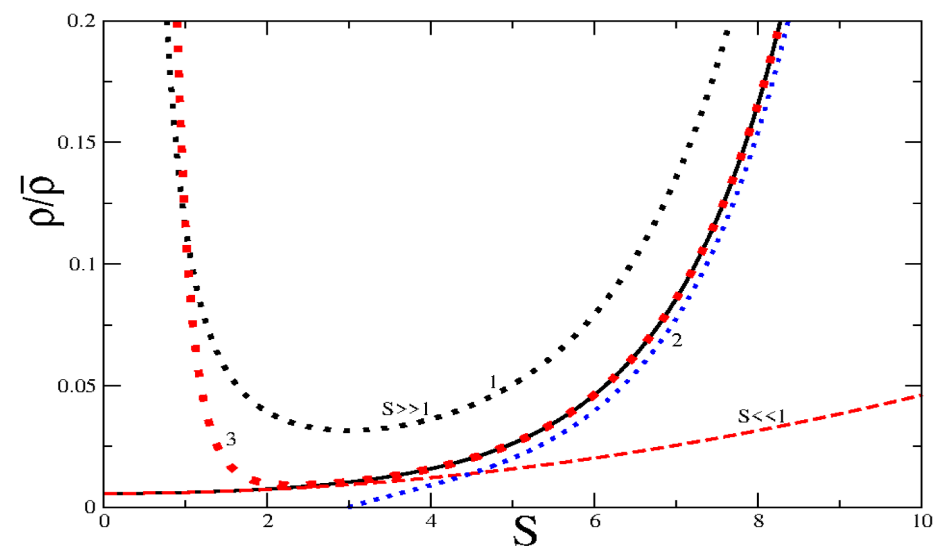

Fig. 1. The MMA level density $\boldsymbol{\rho}$ (solid line in units of $\bar{\boldsymbol{\rho}}$) is shown as function of the entropy $\boldsymbol{S}$ for different approximations: 1) $\boldsymbol{S} \ll \mathbf{1}$ (red dashed line) at quadratic order $\boldsymbol{\rho} \propto \mathbf{1} + \boldsymbol{S}^2/\mathbf{14} + \cdots$; 2) $\boldsymbol{S} \gg \mathbf{1}$ «1», «2», and «3» for expansions, up to zeroth, first, and second-order terms, respectively, in the square brackets of: $\boldsymbol{\rho} \propto \exp(\boldsymbol{S}) \left[\mathbf{1} - \frac{3}{S} + \frac{3}{S^2} \dots\right]$.

Additional integrals of motion can also be the projection of angular momentum M of a nuclear system for rotational of deformed nuclei [2,3]. Fitting the MMA total level density, ρ, for a set of the integrals of motion, $\mathbf{Q} = (N, Z, M)$, to the experimental data for low excitation energy states in a nucleus, one obtains the results for the inverse level-density parameter $K = A/a$, where a, is the level density parameter a, and $A = N + Z$. For the entropy S, one obtains $S = 2(aU)^{1/2}$ where $U = E - E_0 - J\omega^2/2$, $E_0 = E_{\rm ETF} + \delta E$, $E_{\rm ETF}$ is the smooth ETF energy part, and δE is the energy shell correction to the background energy E_0. Similarly, one has $J = J_{\rm ETF} + \delta J$ as the corresponding decomposition for the moment of inertia J. The shell corrections δE and δJ determine the oscillating part δS of the entropy S through the shell corrections $\delta\Omega$ to the generalized grand-canonical potential Ω with a similar decomposition $\Omega = \Omega_{\rm ETF} + \delta\Omega$. Within the semiclassical periodic-orbit theory at thermal equilibrium with temperature T and zero spin, one finds

$$\delta\Omega = \sum_{\rm PO} \left(\hbar^2/t_{\rm PO}^2\right)(\tau_{\rm PO}/\sinh\tau_{\rm PO})\, g_{\rm PO} \cos(S_{\rm PO}/\hbar - \mu_{\rm PO}\pi/2 + {\rm const})\,,$$

where $\tau_{\rm PO} = \pi T t_{\rm PO}/\hbar$; $t_{\rm PO} = \partial S_{\rm PO}/\partial e$ is the period, $S_{\rm PO}(e)$ is the classical action, $\mu_{\rm PO}$ is the Maslov index, and $g_{\rm PO}$ is the single-particle level-density amplitude for the periodic orbit (PO) at the Fermi energy $e = e_F$. For small temperatures T, one obtains $\delta\Omega \to \delta E$. For large temperatures, $T \gtrsim T_{\rm SH} \approx D_{\rm SH}/\pi = (2-3)$ MeV, at large particle numbers $A = 100 - 200$, one finds an exponential decrease of shell effects. Here, the distance between major shells was evaluated semiclassically as $D_{\rm SH} \approx 2\pi\hbar/t_{\rm PO} \approx e_F/A^{1/3} = (7-10)$ MeV. Fig. 2 shows nice agreement of the MMA results [2] for the level density with experimental data for several deformed nuclei at low excitation energies. The MMA at low excitation energies clearly manifests an advantage over the standard Fermi gas asymptote (FG) [1] because of no divergences of the MMA in the limit of small excitation energies U. Another advantage takes place for ^{166}Ho, which has a lot of states in the very low-energy range (cf. Fig. 2(b), and Fig. 2(a)). Shell effects of the MMA approach are important for nuclei ^{166}Ho and ^{240}Pu, in contrast to the nucleus ^{150}Sm.

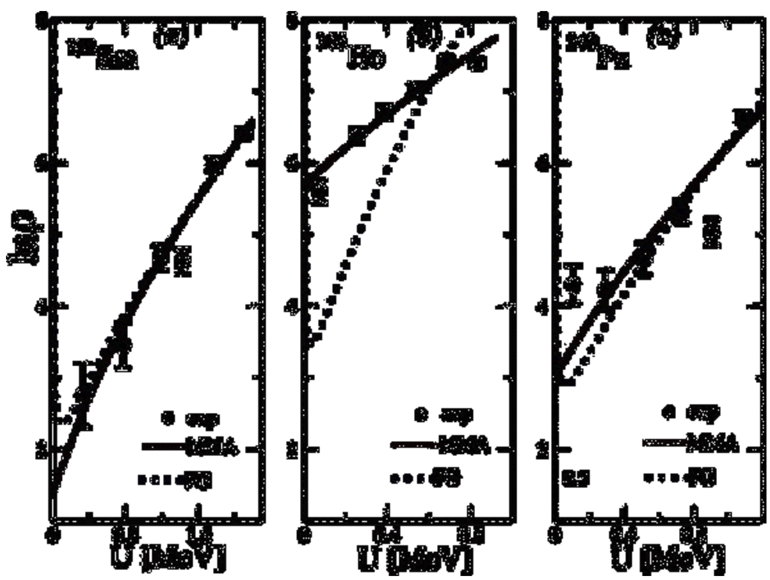

Fig. 2. Level density, $\ln\boldsymbol{\rho}$, as function of the excitation energy $\boldsymbol{U}$ in shown nuclei, obtained in the MMA approach (red solids) for the smallest critical error parameter of the least mean-square fit $\boldsymbol{\sigma}$ [2]. Blue dots present the results of the Fermi gas (FG) approach [1]. Experimental dots are obtained by the sample method from quantum spectra of the ENSDF database http://www.nndc.bnl.gov/ensdf, accounting for spin degeneracies of nuclear states.

The inverse level density parameter, K, was obtained by one-parametric fit of the MMA level densities ρ, taking into account the shell and neutron-proton asymmetry effects, with the experimental results for several chains of isotopes. We have found a significant shell effect in the parameter K as function of the particle number A for the nuclear low-energy states range within the POT. We emphasize the importance of the shell, neutron-proton asymmetry, and rotational effects in these calculations. Taking long Pt and Nd isotope chains as typical examples, one finds a saw-toothed behavior of $K(A)$ as function of the particle numbers A, and its remarkable shell oscillation. We obtained values of K, that are significantly larger than those obtained for neutron resonances, due mainly to accounting for the shell effects. We show that the semiclassical POT is helpful in the low-energy states range for obtaining analytical shell-structure descriptions of the level density. The main part of the interparticle interaction is described in terms of the ETF counterparts of the statistically averaged nuclear potential, and of the level density parameter. Our MMA approach accounting for the spin dependence of the level density was extended to the collective rotations of deformed nuclei within the unified rotation model [2,3]. The well-known effects of the enhancement due to the nuclear collective rotations were found with accounting for the shell structure and neutron-proton asymmetry [3]. This approach might be interesting in the study of isomeric quasistationary states in strongly deformed nuclei at high spins. For perspectives, we suggest also to use our results for collective quantum spectra in deformed rotating nuclei obtained in the two-neutron transfer reactions (p,t), and for calculations of the fission widths. We may apply the MMA approach for metallic clusters and quantum dots, as well as for several problems in nuclear astrophysics.

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